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TECHNICAL REPORT

Empirical Formulas for 14-MeV (*n*, *p*) and (*n*, *α*) Cross Sections

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Empirical formulas for the 14 MeV (*n,p*) and (*n*, *α*) cross sections given by Levkovskii were modified separately in three ranges of mass number, in each of which, coefficients modifying Levkovskii's formulas were determined by least-squares fitting to experimental cross sections. The resulting modified formulas yielded cross sections representing markedly smaller chi-square deviations from experimental values, and moreover gathered closer to unity, compared with calculation using Levkovskii's original formulas.

***KEYWORDS:** comparative evaluations, computer calculations, empirical formula, Levkovskii's formulas, mass number, MeV range 10-100, (n,α) cross section, (n,p) cross section*

I. INTRODUCTION

A number of empirical formulas are available for predicting (*n,p*) and (*n*, *α*) cross sections at 14 MeV[(1)(2)], among which, those proposed by Levkovskii[(2)] are so far probably the most valid, or the most widely used ones. Levkovskii's formulas cover a broad range of mass number with expressions giving the (*n,p*) and (*n*, *α*) cross sections, respectively, as

$$\sigma_{n,p} \propto \sigma_n \exp[-33(N-Z)/A],$$

$$\sigma_{n,\alpha} \propto \sigma_\alpha \exp[-33(N-Z)/A], \qquad (1)$$

where $\sigma_n = \pi r_0^2(A^{1/3}+1)^2$, and $\sigma_\alpha = 0.4\pi r_0^2(A^{1/3}+1)^2$ with $r_0 = 1.2 \times 10^{-13}$ cm.

The (*n,p*) and (*n*, *α*) cross sections derived with these formulas, however, both deviate rather conspicuously from the experimental values for nuclei of a given atomic number Z toward the extremities of mass number A.

The foregoing Levkovskii's formulas have been provided theoretical support by Pai *et al.*[(3)(4)], who compared with statistical-model calculations the experimental 14-MeV (*n,p*) cross sections of nuclei for $32 \leqslant A \leqslant 98$. They obtained better agreement between theory and experiment, by introducing an effective Q-value into the statistical-model calculation. This expedient was justified by considering competition between the (*n,p*) and (*n,n'*) reactions. It was shown[(4)] that Levkovskii's formulas were roughly equivalent to theoretical approximations for 14 MeV neutron cross sections derived on the basis of the effective Q-value[(3)].

For heavy elements, the experimental cross sections of 14-MeV (*n,p*) and (*n*, *α*) reactions are known to exceed in order of magnitude the values[(5)] calculated on the basis of the statistical evaporation model. This suggests[(3)] non-compound process playing a part in these reactions in the case of heavy elements, as evidenced by the dominance of the pre-equilibrium process known to prevail with heavy nuclei[(6)], whereas with intermediate nuclei it is the compound process that dominates[(3)].

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The foregoing circumstances make it difficult to derive a generalized expression that will yield reliable cross sections over the entire scope of mass numbers. In the present study, the scope of mass numbers was divided into three ranges, in each of which, Levkovskii's formulas have been modified by least-squares fitting to experimental values.

II. EMPIRICAL FORMULAS

Levkovskii's formula[2] for the 14-MeV (n,p) cross section is

$$\sigma_{n,p}(mb)$$
$$=45.2(A^{1/3}+1)^2\exp[-33(N-Z)/A), \quad (2)$$

and for the (n,α) cross section

$$\sigma_{n,\alpha}(mb)$$
$$=18.1(A^{1/3}+1)^2\exp[-33(N-Z)/A). \quad (3)$$

The ratio between the experimental and calculated cross sections is plotted against mass number A in **Figs. 1** and **2**, respectively, for the (n,p) and (n, α) reactions. In these figures, the plots for isotopes of a given element are jointed together by solid line. In both figures, the upper diagram represents the experiment-to-calculation ratio for Levkovskii's formulas. For the (n,p) reaction, a distinct trend is seen of decreasing ratio with increasing mass number for a given elements, whereas no such trend is discernible in the case of the (n, α)reaction. Substitution of the coefficients attached to the asymmetry parameter $s=(N\text{-}Z)/A$ in Eqs. (2) and (3) by larger values will serve to attenuate the trend noted above for the (n,p) reaction, but this will on the other hand introduce a new trend for the ratio to decrease consistently with increasing mass number through the entire scope of mass number for all elements. This latter drawback can be removed through replacement of the coefficient $(A^{1/3}+1)^2$ by $A^b(b>2/3)$. Based on the foregoing considerations, Levkovskii's formulas were modified into the form :

$$\sigma=aA^b\exp[-c(N-Z)/A], \quad (4)$$

where the coefficients a, b and c are determined by least-squares fitting, with minimization of

$$\chi^2=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma^i_{\text{exp}}-\sigma^i_{\text{cal}}}{\Delta\sigma^i_{\text{exp}}}\right)^2, \quad (5)$$

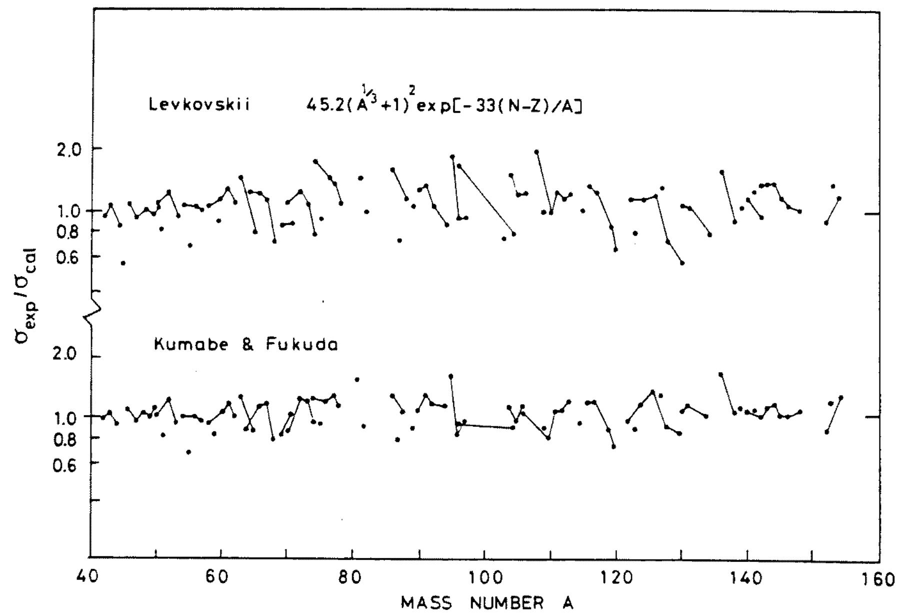

Fig. 1 Ratios of experimental-to-calculated (n,p) cross sections derived using formulas given in Table 1

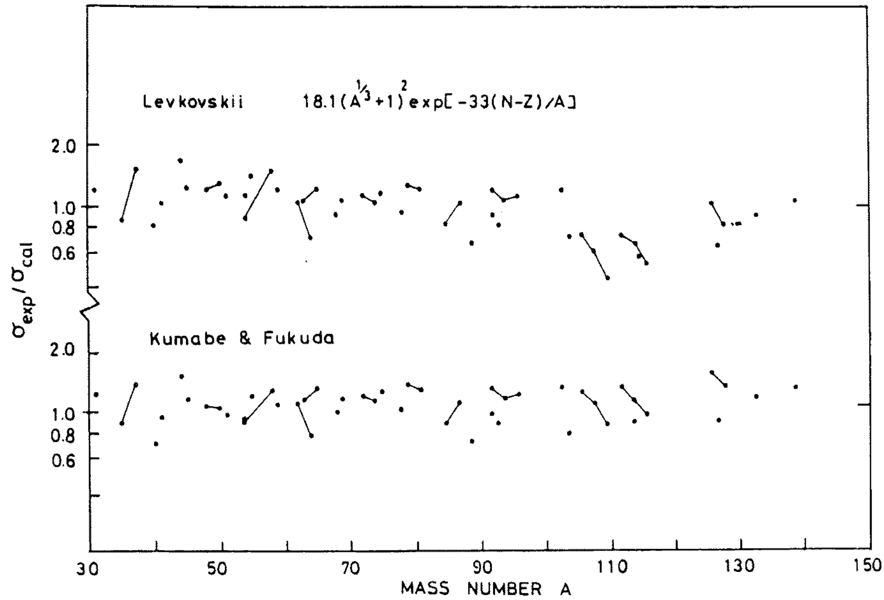

Fig. 2 Ratios of experimental-to-calculated (n, α) cross sections derived using formulas given in Table 3

where σ^i_{exp} and σ^i_{cal} are the experimental and calculated cross sections, respectively, and $\Delta\sigma^i_{exp}$ is the error associated with σ^i_{exp}.

It has already been noted that the compound process is dominant on intermediate nuclei, and the pre-equilibrium process on heavy nuclei. This circumstance led us to divide the overall range of mass number into the three regions indicated in Tables 1 and 3. In the case of the (n,p) reaction, the region I ($A=40\sim62$) is that where the compound process dominates, whereas it is the pre-equilibrium process that dominates in the region III ($A=90\sim160$), while in the intermediate region II ($A=63\sim89$) the reaction is governed by both processes.

Separately for each of the above three regions, the coefficient a, b and c in Eq. (4) were determined by least-squares fitting to the experimental values of cross section critically evaluated by Levkovskii[(2)]. Small changes in the value of b can well be substituted by appropriately varying a without affecting the minimum value of χ^2 to any appreciable extent, and hence for the sake of simplicity, b was altered in steps of integer or of half-integers. The resulting best-fit parameters and χ^2-values obtained for the (n,p) reaction are listed in **Tables 1** and **2**, respectively. The revised formulas thus derived yield the plots shown in the lower diagram of Fig. 1, which reveals almost complete suppresion of the trend seen with Levkovskii's formula of decreasing experiment-to-calculation ratio with increasing mass number for a given element. Table 2 further reveals marked improvement over Levkovskii's formula in terms of χ^2-values, which are seen to have been significantly re-

Table 1 Empirical formulas for 14-MeV (n,p) reaction

	Region	A	Empirical formula (mb)
Levkovskii		40～202	$45.2(A^{1/3}+1)^2\exp[-33(N-Z)/A]$
Present	I	40～ 62	$21.8A\exp[-34(N-Z)/A]$
	II	63～ 89	$0.75A^2\exp[-43.2(N-Z)/A]$
	III	90～160	$0.75A^2\exp[-45.0(N-Z)/A]$

duced, and moreover gathered closer to unity. Similar least-squares fitting was carried out also on the (n,α) reaction, the resulting best-fit parameters and χ^2-values being as listed in **Tables 3** and **4**, respectively, and the experiment-to-calculation ratio as plotted in the lower diagram of Fig. 2. Equally marked improvement is seen of χ^2-values, as indicated in Table 4.

Table 2 χ^2 values for (n,p) reaction

Region	A	Levkovskii	Present
I	40～ 62†	1.42	0.88
II	63～ 89	2.92	1.44
III	90～160	2.31	1.09

† Except for ^{45}Sc

Table 3 Empirical formulas for 14-MeV (n, α) reaction

	Region	A	Empirical formula (mb)
Levkovskii		30～150	$18.1(A^{1/3}+1)^2\exp[-33(N-Z)/A]$
Present	I	30～ 60	$51.0A^{1/2}\exp[-30(N-Z)/A]$
	II	61～105	$55.0A^{1/2}\exp[-33(N-Z)/A]$
	III	106～140	$7.6\times10^{-4}A^3\exp[-40(N-Z)/A]$

Table 4 χ^2 values for (n, α) reaction

Region	A	Levkovskii	Present
I	30～ 60	3.14	1.88
II	61～105	1.25	1.21
III	106～140	8.38	0.58

It must be noted here that the present formulas have ensured better agreement with the experimental data by sacrificing the physical meaning of σ_n and σ_α, which had explicitly figured in the original formulas by Levkovskii.

III. DISCUSSION

As mentioned in Cahp. I, Pai *et al.*[3] improved the agreement between theory and experiment by means of statistical-model calculation through adoption of an effective Q-value, and showed[4] that Levkovskii's formulas roughly corresponded to the theoretical approximations for the 14-MeV neutron cross sections derived using this effective Q-value.

One of the authors[7] has analyzed the measured 14-MeV (n,p) cross sections of nuclei with mass number exceeding 90, applying the pre-equilibrium excition model, and adopting effective Q-values similar to that used by Pai *et al.*[3]. Here, the effective Q-values were derived using a semi-empirical mass formula whose parameters are smooth functions of mass number, free from discontinuity near closed shells. The resulting experimental-to-theoretical cross section ratio proved to be closer to unity for values derived with the effective than with the true Q-values.

Generally speaking, (n,p) and (n, α) cross sections are strongly dependent on their Q-values. If the Q-value decreases smoothly with increasing mass number for a given element, the cross sections also decrease smoothly. The actual experimental (n,p) cross sections decrease smoothly with increasing mass number for a given element[7], despite the large differences in true Q-value between magic and non-magic nuclei, and between even-even and odd nuclei. This behavior can be explained in terms of the effective Q-value.

As mentioned earlier, the (n,p) cross sections derived with the statistical and pre-equilibrium models using the effective Q-values agree well with experimental data. This fact provides additional evidence in support of the present empirical formulas, which moreover are smooth functions of mass number, free from discontinuity near closed shell.

The experimental (n, α) cross section, likewise, has been analyzed in a similar manner by the same author[8] on nuclei with mass number exceeding 100. Experimental-to-theoretical cross section ratio was found appreciably closer to unity when derived using the average Q-value

$$Q_{av}=(Q_t+Q_e)/2, \tag{6}$$

where Q_t and Q_e are respectively the true and effective Q-values. This make it difficult to discuss the theoretical relation between the present empirical formulas and the effective Q-value in the case of the (n,α) cross section.

The present formulas for the 14-MeV (n,p) and (n,α) reactions can still be considered to provide a very useful practical tool for estimating quickly and with relatively good accuracy the cross sections in question, in view of the poor agreement with experiment sometimes provided by calculations based on the statistical and/or pre-equilibrium models.

(Text edited grammatically by Mr. M. Yoshida.)

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